ON THE RIGIDITY THEOREM OF SUSLIN AND VOEVODSKY

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Abstract. The so called rigidity theorem fundamentally initiated by A.Suslin in K-theory has evolved in many versions. The most far-reaching formulation, obtained by Suslin and Voevodsky, asserts an equivalence of categories between effective etale torsion motives over a field k with the derived category of torsion Galois modules over k.

Recently, J. Ayoub succeeded in extending this result to the stable étale motivic homotopy category over any base, a category underlined by Morel as a possible definition for étale motives. The most innovative part of his work, based on earlier results of P.A.Østvær and O.Röndigs, is the avoiding of transfers.

In this talk, I will describe another approach, obtained in collaboration with D.C. Cisinski, to the generalization of the rigidity theorem of Suslin and Voevodsky, which is based on the theory of motivic complexes of Voevodsky. In particular, we are able get localization and cancellation for torsion étale motivic complexes, and also to deal with 2-torsion, which is missed in the approach of Ayoub.

Finally, the consequences for integral h-motives of finite type, the very first theory of Voevodsky, are very strong: they form a complete triangulated formalism in the sense of Grothendieck whose torsion coincides with the formalism of SGA4 and rational part coincides with Beilinson motives. This gives a new insight on nowadays theory of mixed motives and its relation with the tale formalism.

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1. Introduction: Rigidity theorems

The rigidity theorem of Suslin arose from the will of computing the algebraic K-theory of an algebraically closed field. Let us recall the following *Quillen-Lichtenbaum conjecture* (stated by Gersten in [Ger73]):

Conjecture. *Let F be an algebraically closed field of exponential characteristic p. Then for any integer $i > 0$, the K-theory group $K_i(F)$ is divisible and moreover:*

$$K_i(F)_{tor} = \begin{cases} 0 & \text{if } i \text{ is even,} \\ \varinjlim_{m \wedge p = 1} \mu_m(F)^{\otimes, j} & \text{if } i = 2j-1, \end{cases}$$

where $\mu_m(F)$ denotes the group of m-th roots of unity in F.

Date: May 2013.

A striking fact predicted by this conjecture is that the torsion part of the K-groups of an algebraically closed field of characteristic p is independant of the field considered. This remark probably lead Suslin to the following theorem, which is the first form of the rigidity theorem:

(Rig1) *Suslin,* [Sus83].– Let F/F_0 be an extension of algebraically closed fields and n an integer n invertible in F_0. Then:

$$K_*(F_0;\mathbb{Z}/n\mathbb{Z}) \xrightarrow{\sim} K_*(F;\mathbb{Z}/n\mathbb{Z}).$$

In fact, at the time where Suslin proved that theorem, it solved the Quillen-Lichtenbaum conjecture in positive characteristic because of the computation of the K-theory of finite fields due to Quillen (cf [Qui72]). Shortly after obtaining the above result, Suslin had proved the Quillen-Lichtenbaum conjecture in the case of the complex numbers (cf [Sus84]), thus proving it in full generality according to the rigidity theorem (Rig1).

But still, the idea of Suslin continued to evolve in the following variants:

(Rig2) *Gillet-Thomason,* [GT84].– Let k be a separably closed field, R be a strictly henselian regular k-algebra of geometrical type[1], and n be an integer invertible in k. Then:

$$K_*(k;\mathbb{Z}/n\mathbb{Z}) \xrightarrow{\sim} K_*(R;\mathbb{Z}/n\mathbb{Z}).$$

One deduces that for any smooth k-scheme X, the sheaf on $X_{\text{ét}}$ associated with $K_*(-,\mathbb{Z}/n\mathbb{Z})$ is constant, isomorphic to $K_*(k,\mathbb{Z}/n\mathbb{Z})$.

(Rig3) *Gabber,* [Gab92].– Let R be an henselian local ring with residue field k and assume n is an integer invertible in R. Then:

$$K_*(k;\mathbb{Z}/n\mathbb{Z}) \xrightarrow{\sim} K_*(R;\mathbb{Z}/n\mathbb{Z}).$$

Remarque 1.1. All these results make use of transfers and homotopy invariance in K-theory to obtained that a certain action of finite correspondences on $K_*(-;\mathbb{Z}/n\mathbb{Z})$ factors through the Jacobian of a smooth projective curve (or the generalized Jacobian of a smooth curve). Then the main argument is that this action is trivial as the Jacobian is divisible. According to Levine, this principle is first due to Roĭtman to prove his celebrated result on torsion 0-cycles ([Roĭ80]).

These results evolved in the theory of homotopy invariant sheaves with transfers by Voevodsky: the conclusion of the first and second points is valid for any such sheaf F, provided it is a sheaf for the étale topology and it is annihilated by n.

1.2. Let us prepare the statement of the final form of the rigidity theorems.This is an elaboration of (Rig2) above applied to the case of sheaves as in the preceding paragraph.

Put $R = \mathbb{Z}/n\mathbb{Z}$. We fix a perfect field k of finite cohomological dimension and denote by $k_{\text{ét}}$ (resp. $\mathscr{S}m/k)_{\text{ét}}$ the small étale (resp. smooth-étale) site of k.

The natural inclusion $\rho : k_{\text{ét}} \to \mathscr{S}m/k)_{\text{ét}}$ induces an adjunction between, the respective categories of sheaves of R-modules:

$$\rho_\sharp : \mathrm{Sh}(k_{\text{ét}}, R) \leftrightarrows \mathrm{Sh}(\mathscr{S}m/k)_{\text{ét}}, R) : \rho^* \tag{1.2.a}$$

– ρ^* is just the restriction functor. It is a basic fact that $\rho_!$ is a fully faithful and exact functor (comparison between small and big sites). This adjunction can be extended so that we can replace sheaves on the right by sheaves with transfers.

Theorem 1.3 (Voevodsky, [VSF00] Chap. 5, Prop. 3.3.3)**.** *Given the above notations, the induced:*

$$\mathrm{L}\,\rho_\sharp : \mathrm{D}(k_{ét}, R) \leftrightarrows \mathrm{D}(\mathrm{Sh}^{tr}_{ét}(k,R))[W_{\mathbb{A}^1}^{-1}] =: \mathrm{DM}^{eff}_{ét}(k,R) : \mathrm{R}\,\rho^*$$

[1] *i.e.* the strictly local ring of a smooth scheme at some geometrical point.

is an equivalence.

Remarque 1.4. Note in particular, that in this theorem is hidden the computation in $\mathrm{DM}^{eff}_{\text{ét}}(k,R)$:

$$\mathbb{Z}/n(1) \simeq \mu_n.$$

Thus one gets: $\mathrm{DM}^{eff}_{\text{ét}}(k,R) = \mathrm{DM}_{\text{ét}}(k,R)$ (stability with respect to inverse of the Tate twist).

1.5. Let us now describe a generalization, as well as a variant, of this result obtained recently by J. Ayoub.

Let R be a $\mathbb{Z}/n\mathbb{Z}$-algebra. We consider an excellent scheme S such that one of the following conditions holds:

- S is a $\mathbb{Q}$-scheme;
- n is odd.

It is clear that the adjunction (1.2.a) can be extended to the case where S replace $\mathrm{Spec}(k)$.

According to Morel, one can introduce the étale $\mathbb{A}^1$-derived category:

$$\mathrm{D}(\mathrm{Sh}_{\text{ét}}(S,R))[W_{\mathbb{A}^1}^{-1}]$$

as well as its stabilization with respect to the Tate twist which we denote by $\mathbf{DA}^{\text{ét}}(S,R)$ following Ayoub.

Then Ayoub proves:

Theorem 1.6 (Ayoub, [Ayo13])**.** *Given the above notations, the induced adjunction*

$$\mathrm{L}\,\rho_! : \mathrm{D}(S_{\acute{e}t},R) \leftrightarrows \mathbf{DA}^{\acute{e}t}(S,R) : \mathrm{R}\,\rho^*$$

is an equivalence.

The consequences of this theorem for the theory of étale motives are tremendous. I will mention the principal ones in the followings.

Remarque 1.7. The main technical innovation of this theorem is the use of some weak transfers in stable $\mathbb{A}^1$-homotopical sense – this is where we have to work in a stable world. This is made possible by the work of Ayoub on the theory of Voevodsky's theory of cross functors and the theory of Grothendieck 6 operations. One should also mention two earlier work on this variant of the rigidity theorem:

- S. Yagunov, [Yag04];
- O. Röndigs and P.A. Østvær, [RØ08],

which prove the analog of (Rig1) respectively for representable $\mathbb{A}^1$-cohomology and the whole of the stable $\mathbb{A}^1$-homotopy category.

One should also point out that the proof of Ayoub relies on [RØ08].

2. Torsion étale motivic complexes

2.1. Fix: S a noetherian scheme, n an integer, R a $\mathbb{Z}/n$-algebra, and put $\Lambda = \mathbb{Z}[p^{-1}, p \wedge n = 1]$.

Recall that given smooth S-schemes X and Y, a finite Λ-correspondence from X to Y is a relative cycle on $X \times_S Y$ over X whose support is finite equidimensional over X and which has *good specializations*[2] at any point of X.[3] They have various good properties, and in particular a composition product.

[2] *i.e.* pullbacks

[3] The definition of "specialization" and "good specialization" of cycles is defined in the *theory of relative cycles* by Suslin and Voevodsky; see [VSF00, chap. 2] and [CD09, section 8] for another account.

Then one defines the category $\mathrm{Sh}^{tr}_{\text{ét}}(S,R)$ of *étale R-sheaves with transfers* has the étale R-sheaves over $\mathscr{S}m/S$ equipped with an action of finite Λ-correspondences, compatible with composition.

The first good property of these objects is the following:

Proposition 2.2. *The following functor:*

$$\rho_\sharp : \mathrm{Sh}(S_{\acute{e}t}, R) \to \mathrm{Sh}_{\acute{e}t}(S,R) \xrightarrow{add\ transfers} \mathrm{Sh}^{tr}_{\acute{e}t}(S,R),$$

left adjoint to the obvious restriction functor $\rho^* : F \mapsto F|_{X_{\acute{e}t}}$, is exact and fully faithful.

In other words, étale R-sheaves coming from the small site of S uniquely admits transfers.[4]

The second good property of $\mathrm{Sh}^{tr}_{\text{ét}}(S,R)$ is that it is well behaved, as its Nisnevich analog: in particular, it is a Grothendieck abelian category with a closed monoidal structure. Thus, one can define the category of étale effective motives:

Definition 2.3. One defines the category of étale effective R-motives as the $\mathbb{A}^1$-localization of the derived category:

$$\mathrm{DM}^{eff}_{\text{ét}}(S,R) := \mathrm{D}\left(\mathrm{Sh}^{tr}_{\text{ét}}(S,R)\right)[W_{\mathbb{A}^1}^{-1}].$$

The advantage of using transfers mainly reside in the following proposition:

Proposition 2.4. *Assume n is invertible in S. Then the Tate motive*

$$R(1) := Cone\big(R^{tr}_S(\{1\}) \to R^{tr}_S(\mathbb{G}_m)\big)$$

is isomorphic to $\mu_n \otimes R$ where μ_n is the sheaf of n-th root of unity with its natural transfers (cf Prop. 2.2).

The consequences of this simple proposition, obtained by an extension of the computation of Suslin and Voevodsky, are surprisingly strong:

Corollaire 2.5. *Assume $n > 0$:*

1. *Cancellation.–* $\mathrm{DM}^{eff}_{\acute{e}t}(S,R) \simeq \mathrm{DM}_{\acute{e}t}(S,R)$.
2. *Orientation and purity.– For any smooth projective morphism $p : X \to S$ of dimension d, one has: $p_* \simeq p_\sharp(-d)|-2d]$.*
3. *Localization.– For any closed immersion i with complementary open immersion j, the following triangle is homotopy exact:*
$$j_!j^! \to 1 \to i_*i^*.$$

The first point and the orientation properties are obvious from the preceding proposition. Then the purity property comes from the fact the motive $R^{tr}_S(X)$ is strongly dualizable with an explicite duality with $R^{tr}_S(X)(-d)[-2d]$ (construction of Gysin morphisms and an argument of Ayoub). The localization property comes from our first failed attempt to prove localization for integral Nisnevich motives: it fundamentally relies on (1) and (2).

With these notations, and based on the previous corollary, we can deduce the following theorem (see [CD13, Th. 4.5.5]):

Theorem 2.6. *S noetherian, $S' = S[n^{-1}]$: the adjunction $(\rho_!, \rho^*)$ of Proposition 2.2 induces an equivalence of categories:*

$$\mathrm{L}\,\rho_\sharp : \mathrm{D}(S'_{\acute{e}t}, R) \leftrightarrows \mathrm{DM}^{eff}_{\acute{e}t}(S,R) \simeq \mathrm{DM}_{\acute{e}t}(S,R) : \mathrm{R}\,\rho^*.$$

[4]Note this result is valid even in the case $n = 0$, $\Lambda = \mathbb{Z}$.

Note that Ayoub was also able to deduce this result, but only in the stable case, and assuming that S is normal, excellent, universally japanese and the assumption of 1.5 holds.

Our proof goes as follows: it is not difficult to deduce from Proposition 2.2 and the homotopy invariance of étale cohomology (for any coefficients of $\mathrm{D}(S'_{\text{ét}}, R)$) that $\mathrm{L}\,\rho_!$ is fulyl faithful. Then the key point of is to show that the functor $\mathrm{L}\,\rho_!$ commutes with f_* for a proper morphism f (one uses the proper base change theorem in the context of $\mathrm{D}(S'_{\text{ét}}, R)$ and $\mathrm{DM}^{eff}_{\text{ét}}(S, R)$).

2.7. For the first corollary of the previous theorem, we recall that a complex K of sheaves with transfers over a base scheme S is said to be $\mathbb{A}^1$-*local* if its Nisnevich cohomology on smooth S-schemes is homotopy invariant. Note this definition also applies to sheaves, seen as complexes concentrated in degree 0.

The following corollary is in fact a generalization of the properties (Rig2) and (Rig3) stated in the first section, as well as an extension of the theory of étale homotopy invariant sheaves with transfers over a perfect field of Voevodsky:

Corollaire 2.8. *Assume n is invertible on S.*

1. *Let F be an étale R-sheaf with transfers over S. Then the following conditions are equivalent:*
   - (i) *F is $\mathbb{A}^1$-local;*
   - (ii) *the canonical map $\rho_!\rho^*(F) \to F$ is an isomorphism* – i.e. *F comes from the small site of S.*
2. *More generally, given any complex K of étale R-sheaves with transfers over S, the following conditions are equivalent:*
   - (i) *K is $\mathbb{A}^1$-local;*
   - (ii) *for any integer $i \in \mathbb{Z}$, the cohomology sheaves $\underline{H}^i(K)$ are $\mathbb{A}^1$-local;*
   - (iii) *the canonical map $\rho_!\rho^*(K) \to K$ is an isomorphism* – i.e. *K comes from the small site of S.*

2.9. The previous theorem also has an interesting repercussion on the notion of traces in the context of SGA4, relative to a finite surjective morphism $f : X \to S$.

In [SGA4, XVII, sec. 6.2], these traces where defined in the case where f is flat or more generally "pondéré". Our assumption relies on the theory of relative cycles of Suslin-Voveodsky: we say that the morphism $f : X \to S$ is Λ-*universal* if the fundamental cycle associated with X is a relative Λ-cycle over S in the sense of Suslin and Voevodsky (see [CD13, Par. 5.6.4], or [CD09, 8.1.48]).

Examples of such morphisms are:

- f is flat;
- the aim S of f is regular;
- the aim S of f is geometrically unibranch and has residue fields whose exponential characteristic is invertible in Λ.

As a corollary of the prevous theorem we obtain:

Corollaire 2.10. *Assume n is invertible on S.*

Let $f : X \to S$ be a finite surjective and Λ-universal morphism. Then for any complex K of R-sheaves on $S_{\text{ét}}$, there exists a trace map:

$$\mathrm{Tr}_f : f_! f^*(K) \to K$$

which is compatible with composition, base change, and satisfies the degree formula (see [CD13, 5.6.8] *for details).*

Remarque 2.11. It is also possible to extend this notion of trace by replacing the assumption finite by separated and quasi-finite as in [SGA4, XVII] (see [CD13, Rem. 5.6.9]).

3. h-motives

Introduction.

3.1. Recall that when R is a $\mathbb{Q}$-algebra, given a base scheme S one has the following equivalences proved in [CD09]:

$$\begin{aligned}
\mathrm{DM}_{\text{ét}}(S,R) &\simeq \mathrm{DM}_{\mathrm{Nis}}(S,R) \ (S \text{ geom. unibranch}), && \text{introduced by Voevodsky and later Cisinski-D.}\\
&\simeq \mathrm{DM}_{\mathrm{B}}(S,R) && \text{introduced by Cisinski-D.,}\\
&\simeq \mathbf{DA}^{\text{ét}}(S,R) && \text{introduced by Morel, Ayoub,}\\
&\simeq \mathrm{DM}_h(S,R) && \text{(from the) original construction of Voevodsky.}
\end{aligned}$$

The profusion of good models for rational mixes motives can be puzzling. On the other hand, we know for sure that they are equivalent.

Moreover, in each of these cases, one can derive the full formalism of Grothendieck 6 operations, and using the torsion case described earlier, one can extend this to the integral case for:

- $\mathrm{DM}_{\text{ét}}(-,\mathbb{Z})$, but one has to restrict to excellent geometrically unibranch schemes;
- $\mathbf{DA}^{\text{ét}}(-,\mathbb{Z}[1/2])$, this is done by Ayoub as a corollary of its rigidity theorem: see [Ayo13, Sec. 7, 8].

However, there is a model without defect, and this is the original one, introduced by Voevodsky. This is what I want to describe finally.

3.2. Recall the definition of the h-topology on the category of S-schemes of finite type: a covering for this topology is a morphism of S-schemes

$$p : W \to X$$

such that p is a universal topological epimorphism – the topology on the target is the image topology of that of W, and this property remains true after any base change $X' \to X$ (eg.: proper surjective, faithfully flat).

Given any ring R, let $\mathrm{Sh}_{\mathrm{h}}(S,R)$ be the category of h-sheaves of R-modules on $\mathscr{S}_S^{ft}$. Stabilizing the original definition of Voevodsky, we define:

Definition 3.3 (Voevodsky)**.** Given the notations above, we define the following categories:

- the (big) category of effective h-motives $\underline{\mathrm{DM}}_{\mathrm{h}}^{\mathit{eff}}(S,R)$ as the $\mathbb{A}^1$-localization of the derived category of $\mathrm{Sh}_{\mathrm{h}}(S,R)$;
- the (big) category of h-motives $\underline{\mathrm{DM}}_{\mathrm{h}}(S,R)$ as the $\mathbb{P}^1$-stabilization of the homotopy category $\underline{\mathrm{DM}}_{\mathrm{h}}^{\mathit{eff}}(S,R)$;
- the category of *étale motives* $\mathrm{DM}_{\mathrm{h}}(S,R)$ as the full triangulated subcategory of $\underline{\mathrm{DM}}_{\mathrm{h}}(S,R)$ generated by the motives of the form $R_S^{\mathrm{h}}(X)(n)$ for a *smooth* S-scheme X and an integer $n \in \mathbb{Z}$ and stable by infinite direct sums;
- the category of *constructible étale motives* $\mathrm{DM}_{\mathrm{h},c}(S,R)$ as the full triangulated thick subcategory of $\underline{\mathrm{DM}}_{\mathrm{h}}(S,R)$ generated by the motives of the form $R_S^{\mathrm{h}}(X)(n)$ for a *smooth* S-scheme X and an integer $n \in \mathbb{Z}$.

Remarque 3.4. Voevodsky has used the terminology "of finite type", and later "geometrical", for what we call here "constructible".

This very abstract construction still enjoys the following incredible properties:

Theorem 3.5. *Let S be an excellent scheme and R a commutative ring.*

(1) *If R is a $\mathbb{Q}$-algebra, $\mathrm{DM}_{h,c}(S,R)$ is the category of rational, constructible, mixed motives we all know (cf above).*

(2) *Let $n > 0$ be an integer and let $S' = S[1/n]$ be the maximal open subscheme of S on which n is invertible. If R is a $\mathbb{Z}/n$-algebra then*

$$\mathrm{DM}_{h,c}(S, R) \simeq \mathrm{D}^b_c(S', R),$$

the later category being the category of bounded complexes sheaves on $S_{\acute{e}t}$ with constructible cohomology of SGA4.

In the case where S is a (perfect) field, these results are due to Voevodsky.

One can deduce from this theorem that the category $\mathrm{DM}_{\mathrm{h}}(S, \mathbb{Z})$ satisfies all the good properties of the categories appearing in (1) and (2): Grothendieck 6 operations for excellent schemes and the duality formalism (see [CD13, sec. 5.8]).

3.6. We finish this abstract with another construction that one can derive from the previous theorem, the notion of l-completion of h-motives.

Fix a prime integer l. Given any étale integral motive K in $\mathrm{DM}_{\mathrm{h}}(S, \mathbb{Z})$, one can define its homotopical l-completion:

$$\hat{K}^l = \mathrm{R}\varprojlim_{n>0} K/l^n$$

where $K/l^n := K \otimes^{\mathrm{L}}_{\mathbb{Z}} \mathbb{Z}/l^n\mathbb{Z}$.

One says that K is *l-complete* if the natural map $K \to \hat{K}^l$ is an isomorphism. Then, we say that K is *l-constructible* if K/l is constructible.

One defines the l-adic completion $\mathrm{DM}_{\mathrm{h},c}(S, \mathbb{Z})^{\wedge_l}$ of $\mathrm{DM}_{\mathrm{h},c}(S, \mathbb{Z})$ as the full subcategory made by the l-complete and l-constructible étale motives. According to point (2) of the above theorem, one gets an equivalence of triangulated categories:

$$\mathrm{DM}_{\mathrm{h},c}(S, \mathbb{Z})^{\wedge_l} \to \mathrm{D}^b_c(S, \mathbb{Z}_l)$$

where the right hand side category is the category of l-adic systems introduced by Ekedhal when S is separated of finite type over a regular scheme of dimension less than 1, and l is invertible on S.

Then one gets a natural realization functor:

$$\mathrm{DM}_{\mathrm{h},c}(S, \mathbb{Z}) \to \mathrm{DM}_{\mathrm{h},c}(S, \mathbb{Z})^{\wedge_l} \to \mathrm{D}^b_c(S, \mathbb{Z}_l), K \mapsto \hat{K}^l$$

which is simply an l-completion functor. By looking at the rational part of this functor, one deduces a rational realization functor which, according to proof of Ayoub, is the dual of the one already constructed by Ivorra ([Ivo07]).

Now recall that the analog functor:

$$\mathrm{D}_{perf}(\mathbb{Z}-\mathrm{mod}) \to \mathrm{D}_{perf}(\mathbb{Z}_l-\mathrm{mod}), K \mapsto \hat{K}^l,$$

where *perf* means we are considering the full subcategories made by perfect complexes on both sides, is conservative when restricted to the rational parts. Thus one can reformulate the conservativity conjecture by the question:

Is the property of constructibility for integral étale motives analog to that of perfect complexes for abelian groups ?

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